Research Article

Developing a 1D kinematic wave model for simulating the downstream flow of Tra Khuc river

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Abstract: The nonlinear kinematic wave model is developed from the Saint Venant system of equations, which includes a nonlinear kinematic wave program that solves the system of equations by Newton's iterative method and a linear kinematic wave program for calculations initial flow value. The developed model is tested with sample problems and compared with the simulation results by Mike 11 model on Tra Khuc river. Evaluating the simulation results of these two models show that, the simulation results of the Mike 11 model are better than the kinematic wave model, not significantly in the upstream and midstream flow, but significantly in the downstream flow of the Tra Khuc river. The simultaneous results show that the 1−dimensional kinetic wave model has sufficient reliability and applicability.

Keywords: Mike 11 model; Kinematic Wave Model; Tra Khuc River.

1. Introduction

Flow analysis models play an important role in simulation and forecasting of river flow, so it is formed very early. Simple models such as SSARR, which uses the method of reserves of river sections, or Muskingum model which simulates the displacement of flood waves on a linear channel, do not simulate the motion of the flow. More general research from Cunge (1969), where the discrete Muskingum equation is considered to be a finite difference approximation of the kinematic wave model. Therefore, the Muskingum–Cunge model is a kinematic wave approximation model [1].

Hydraulic models are widely used and are quite popular today. Depending on the difference method and assumptions, there are different hydraulic models, such as: MIKE 11, HEC-RAS, KOD1, VRSAP, ISIS-1D, HydroGIS, MK4, QUAL2-E, HEC 6, IMECH-1D, DUFLOW. The dynamic wave model has a wide range of applications on tidal and non-tidal affected rivers, with good simulation quality. However, this type of model uses the river cross-section as the input, so it cannot be applied on the river section without cross-section, while lack of cross-section is quite common in upstream rivers. Therefore, the application of models capable of simulating physical nature of flow but not requiring cross-sectional data is essential. To overcome the above problem, kinematic wave model is studied and applied;

in which, some simulation components are reduced compared to the dynamic wave model, but do not use cross-sectional data. The kinematic wave model is based on a simplified form of Saint Venant's equations [2], a dynamic wave approximation [4], which simulates the process of flood propagation in rivers due to changes in discharge or water level. In addition, the simulation principle of the kinematic wave model is that the flow is generated from the slope, so it is suitable for mountainous rivers and streams, where commonly lack of cross-sectional data.

In theory, the kinematic wave model is simpler, requires less data, so the application scope and simulation quality are lower than the dynamic wave model. However, this issue needs to be verified in practice. There are many dynamic wave models today, but Mike 11 is the most commonly used. Therefore, this research evaluates the simulation ability of the kinematic wave model and the Mike 11 model.

Kinematic wave model is proposed by [3] and developed by a number of later researches, which is applied to simulate flow in canals and rivers [4–8]. The one-dimensional model in the river is later studied by [9] for riverbanks with different types of cross-sectional shapes. Nwaogazie (1978) developes a nonlinear one-dimensional model by Newton's method Raphson [10], [11] builds the model in the channel by solving finite difference. Before, the kinematic wave model is developed by many researchers to simulate flow on slopes such as: [12, 13] calculates the peak transmission time between hydrological stations, while Morel [14] builds a model combining hydraulics and statistics. In addition, the kinematic wave model is also used in the Mike-11, HEC-1 model to be used for rivers without cross-sectional data. However, these models only simulate a river tributary, using the matrix method to solve the problem. In Vietnam, the one-dimensional model on the slope has been used by [15] to simulate the slope flow in the KW1D model.

The river section applied to evaluate the simulation ability of the kinematic wave model and Mike 11 model is Tra Khuc river, Quang Ngai province. This is a large river basin of the province, where rain and flood events are very complicated. Especially, the river basin is near Ba To - a heavy rain center of the country, so floods occur harshly. The climatic changes make the flow characteristics in the river basin become complicated. In the dry season of 2015, while the central region, including the Tra Khuc river basin, deals with a severe drought and lack of water, an unusual flood occurs suddenly on March 27. This flood isolates more than 400 households with 1,000 people and submerges 100 hectares of watermelons in water. The flood in November 2009 kills 26 people, injures 6 people, destroys 10,430 houses, and causes total damage of 47.66 billion VND. After that, the flood in December 2009 makes 41 people died, 11 people missing, 21 people injured, with a total loss of 200 billion VND. The historic flood in November 2013 kills 40 people, hundreds of houses are washed away, and damage is estimated at 179 billion VND. The extremes of the flow, the fierce nature of rain and floods occur with more frequency, greater intensity and more complex development, requiring more advanced forecasting work. Therefore, it is necessary to study and apply new models and technologies for flood forecasting and water resources calculation on Tra Khuc river.

2. Materials and methods

2.1. Developed model

The Saint Venant equation has many different simplifications, each of them defines a one-dimensionally distributed unstable flow simulation model. Continuity equation, conservative momentum equation and non-conservative momentum equation neglecting lateral currents, wind resistance, and eddy losses are used to define different types of models for one-dimensionally distributed unstable flow [2, 3−9].

The equation of momentum includes the components of the physical processes that control the flow of momentum. These components are: the local acceleration component

which describes the change in momentum due to the change of velocity with time; the convective acceleration component which describes the change in momentum caused by the change in velocity along the channel; the pressure component which is proportional to the change in water depth along the channel; the gravity component which is proportional to the bottom slope S0; and the friction component which is proportional to the friction gradient Sf. The components of local acceleration and convective acceleration represent the effects of inertial forces on the flow [1−2, 16−18].

+ Continuity equation:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{1}$$

+ Momentum equation [1]:

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0 \tag{2}$$

Kinematic waves dominate the flow when the forces of inertia and pressure is neglected. In kinematic waves, the forces of friction and gravity are balanced, so the water flow has no acceleration. Therefore, the energy line is parallel to the bottom of the channel and the flow in an elemental segment is a steady stream (because $S_0 = S_f$).

Kinematic waves are caused by changes in flow such as changes in water flow or wave speed which is the changing velocity along the channel. The wave speed depends on the type of wave and it can be completely different from the water velocity. For kinematic waves, the acceleration and pressure components in the momentum equation have been neglected, so the wave motion is described mainly by the continuity equation. That is why it is called kinematic wave, because kinematics studies motion in which the influence of mass and force is not taken into account. The kinematic wave model is determined by the following equations [23−26]:

+ Continuity equation:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{3}$$

+ Momentum equation:

$$S_o = S_f \tag{4}$$
$$A = \alpha Q^\beta \tag{5}$$

In the Manning equation, $S_o = S_f$ và R=A/P, therefore:

$$Q = \frac{1.49 S_o^{1/2}}{n P^{2/3}} A^{5/3} \tag{6}$$

Rewrite equation (6) for A, from which to find α and β = 0.6 as follows [1]:

$$A = \left(\frac{n P^{2/3}}{1.49\sqrt{S_o}}\right)^{3/5} Q^{3/5} \tag{7}$$

$$A = \left(\frac{n P^{2/3}}{1.49\sqrt{S_o}}\right)^{0.6} \tag{8}$$

Equation (1) depends only on A and Q, where A is defined in equation (5). The partial derivative of equation (5) of variables A and Q with respect to t and then substituting into equation (1) to get equation (9). Substitute equation (9) into equation (3) to get equation (10). Equation (10) is converted into difference form diagrammatically linear according to equation (17), and diagrammatically nonlinear according to equation (22).

$$\frac{\partial A}{\partial t} = \alpha\beta Q^{\beta-1}\left(\frac{\partial Q}{\partial t}\right) \tag{9}$$

$$\frac{\partial Q}{\partial x} + \alpha\beta Q^{\beta-1}\left(\frac{\partial Q}{\partial t}\right) = q \tag{10}$$

2.3. *Developing a 1D kinematic wave model*

The model is developed on Fortran 90 programming language and consists of two main parts: linear and nonlinear kinematic wave model. In which the linear model is used as the first solution of the nonlinear model. The linear model is solved by the hidden difference diagram, the nonlinear model is solved by the Newton iterative method [2].

2.3.1. Linear kinematic wave diagramD setup

Apply the hidden difference diagram [1−2]:

$$\frac{\partial u_{i+1}^{j+1}}{\partial x} = \frac{u_{i+1}^{j+1} - u_i^{j+1}}{\Delta x} \tag{11}$$

$$\frac{\partial u_{i+1}^{j+1}}{\partial t} = \frac{u_{i+1}^{j+1} - u_{i+1}^j}{\Delta t} \tag{12}$$

$$\frac{\partial Q_{i+1}^{j+1}}{\partial x} \approx \frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} \tag{13}$$

$$\frac{\partial Q_{i+1}^{j+1}}{\partial t} \approx \frac{Q_{i+1}^{j+1} - Q_{i+1}^j}{\Delta t} \tag{14}$$

$$Q \approx \frac{Q_i^{j+1} + Q_{i+1}^j}{2} \tag{15}$$

$$q \approx \frac{q_{i+1}^{j+1} + q_{i+1}^j}{2} \tag{16}$$

Substituting the equations from (13) to (16) into the equation (10) to get the linear kinematic wave difference equation and the hidden plot as shown in Figure 1 [1−2]:

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \alpha\beta \left(\frac{Q_{i+1}^j + Q_i^{j+1}}{2}\right)^{\beta-1} \left(\frac{Q_{i+1}^{j+1} - Q_{i+1}^j}{\Delta t}\right) = \frac{q_{i+1}^{j+1} + q_{i+1}^j}{2} \tag{17}$$

$$Q_{i+1}^{j+1} = \frac{\left[\frac{\Delta t}{\Delta x} Q_i^{j+1} + \alpha\beta Q_{i+1}^j \left(\frac{Q_{i+1}^j + Q_j^{j+1}}{2}\right)^{\beta-1} + \Delta t \left(\frac{q_{i+1}^{j+1} + q_{i+1}^j}{2}\right)\right]}{\left[\frac{\Delta t}{\Delta x} + \alpha\beta \left(\frac{Q_{i+1}^j + Q_i^{j+1}}{2}\right)^{\beta-1}\right]} \tag{18}$$
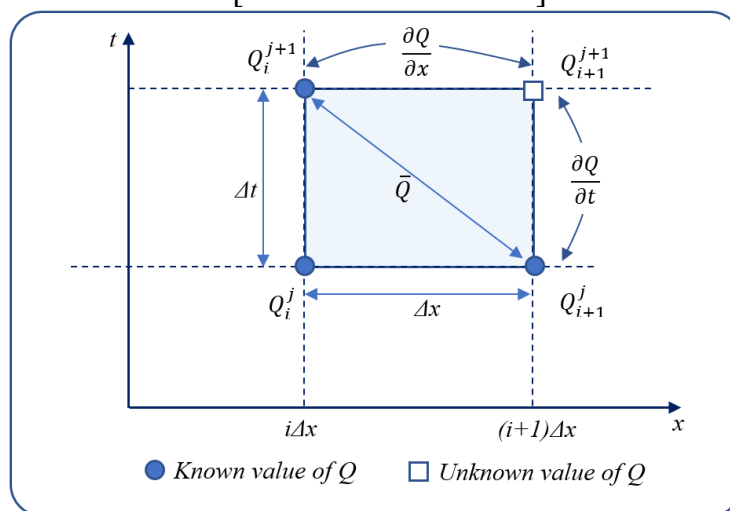

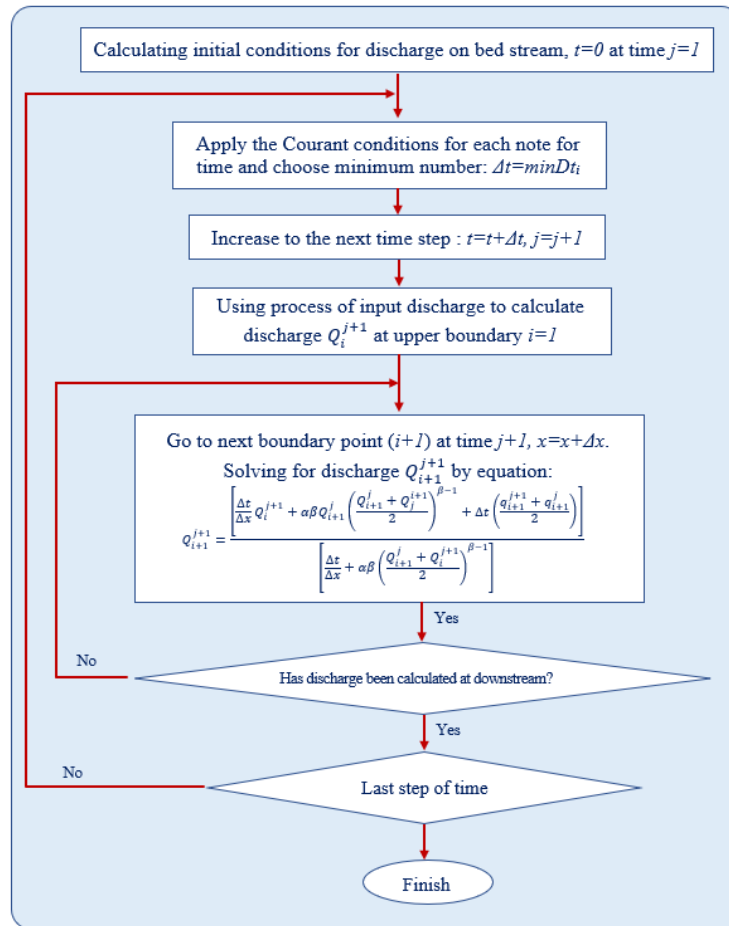**Figure 1.** Hidden difference diagram solving linear kinematic wave equation.

Figure 2. Linear kinematic wave calculation block diagram.

2.3.2. Nonlinear kinematic wave diagram

Equation (10) is transformed into the difference equation [1−2]:

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{A_{i+1}^{j+1} - A_{i+1}^{j}}{\Delta t} = \frac{q_{i+1}^{j+1} + q_{i+1}^{j}}{2} \tag{19}$$

$$A_{i+1}^{j+1} = \alpha\left(Q_{i+1}^{j+1}\right)^{\beta} \tag{20}$$

$$A_{i+1}^{j} = \alpha\left(Q_{i+1}^{j}\right)^{\beta} \tag{21}$$

Substituting equations (20) and (21) into (19):

$$\frac{\Delta t}{\Delta x}Q_{i+1}^{j+1} + \alpha\left(Q_{i+1}^{j+1}\right)^{\beta} = \frac{\Delta t}{\Delta x}Q_i^{j+1} + \alpha\left(Q_{i+1}^{j}\right)^{\beta} + \Delta t\left(\frac{q_{i+1}^{j+1} + q_{i+1}^{j}}{2}\right) \tag{22}$$

This equation is sorted so that the unknown flow Qi+1j+1 is on the left-hand side and the other known quantities are on the right-hand side. This is a nonlinear equation for $Q_{i+1}j+1$ so it needs to be solved numerically. The block diagram below applies Newton's iterative method. The linear model which is developed into the nonlinear model is represented in the initial estimation block using the linear estimator 20 as shown below in Figure 3.
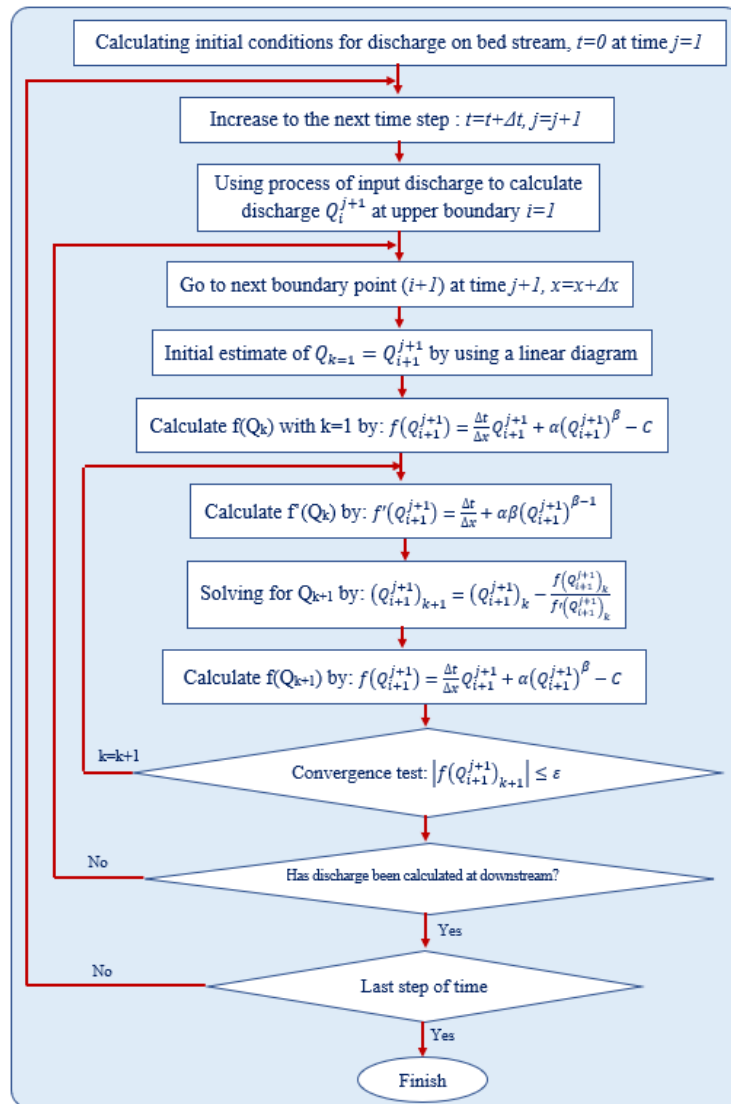
Figure 3. Block diagram for calculating nonlinear kinematic waves.

$$C = \frac{\Delta t}{\Delta x}Q_i^{j+1} + \alpha(Q_{i+1}^j)^\beta + \Delta t\left(\frac{q_{i+1}^{j+1} + q_{i+1}^j}{2}\right) \tag{23}$$

Then a residual error f(Q$_i$+1$_j$+1) is determined by equation (22).

$$f(Q_{i+1}^{j+1}) = \frac{\Delta t}{\Delta x}Q_{i+1}^{j+1} + \alpha(Q_{i+1}^{j+1})^\beta - C \tag{24}$$

The first derivative of f(Q$_i$+1$_j$+1) is as follows:

$$f'(Q_{i+1}^{j+1}) = \frac{\Delta t}{\Delta x} + \alpha\beta(Q_{i+1}^{j+1})^{\beta-1} \tag{25}$$

The goal is to find Q$_i$+1$_j$+1 to force f(Q$_i$+1$_j$+1) to zero. Using Newton's iterative method and the iteration steps k = 1, 2, 3, ...

$$(Q_{i+1}^{j+1})_{k+1} = (Q_{i+1}^{j+1})_k - \frac{f(Q_{i+1}^{j+1})_k}{f'(Q_{i+1}^{j+1})_k} \tag{26}$$

The convergence criterion for the iterative process is:

$$\left|f(Q_{i+1}^{j+1})_{k+1}\right| \le \varepsilon \tag{27}$$

Estimating the initial value of Q$_i$+1$_j$+1 in each iteration has an important influence on the convergence of the diagram. One approach is to use the solution of the linear diagram,

Equation (18) as the first approximation of the nonlinear diagram. Li Simons and Stevens (1975) [1], after conducting stability analysis, showed that the diagram using equation (22) is an unconditional stability scheme and can use values of Δ_t/Δ_x over a fairly wide range without creating large errors in the shape of the discharge process curve.

The model after programming is tested with data in example 9.6.1 in the Applied Hydrology textbook by Vente Chow [1]. The results of the model coincide with the calculation results of the above example, the calculated correlation is shown in Figure 4.
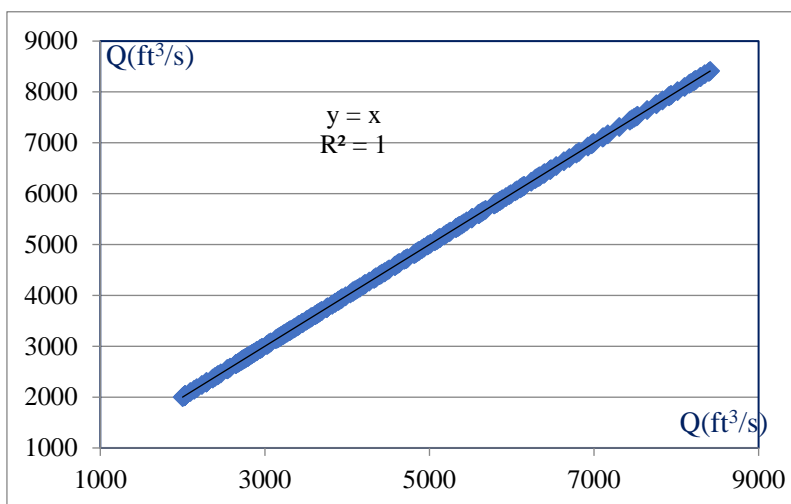
Figure 4. Correlation of results calculated by program and by Vente Chow.

3. Results and discussion

3.1. Models set up for flow simulation on Tra Khuc river

Input data for the Mike 11 and kinematic wave models are taken at the Son Giang hydrological station. The joining boundaries of the To, Son Thanh, Tam Rao, Ham Giang rivers; the position of the boundaries is shown in the model hydraulic diagram Mike 11 (Figures 6 and 8). The study uses floods that occurred from November 22 to 27, 2011 and from September 13 to 15, 2013 in the Tra Khuc river basin.
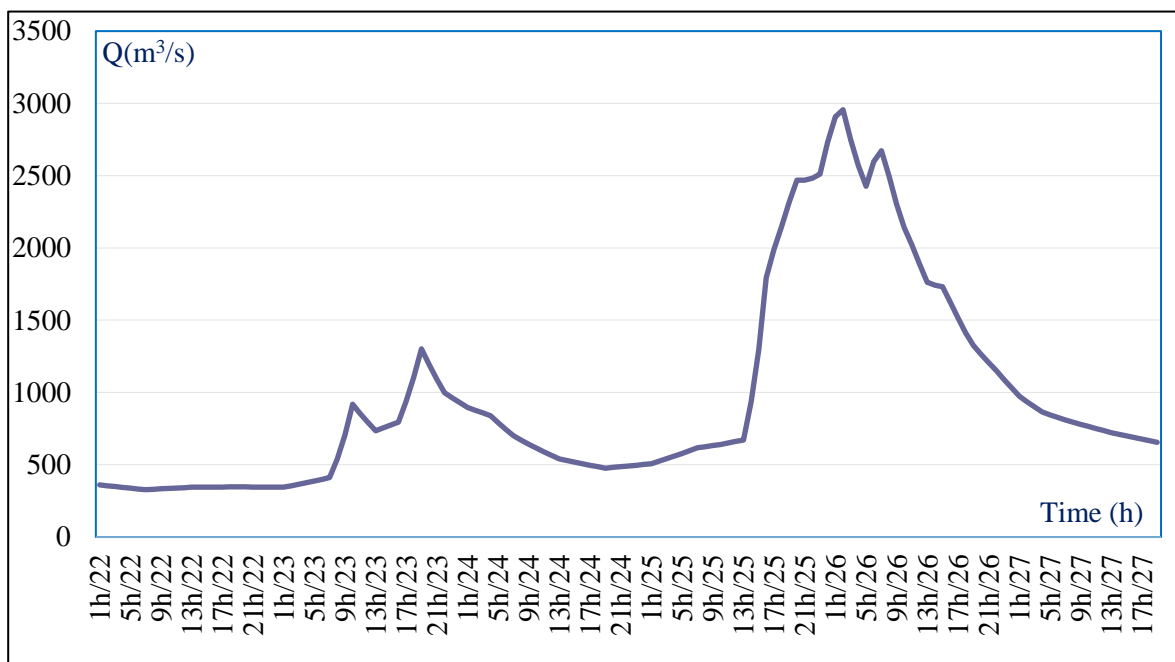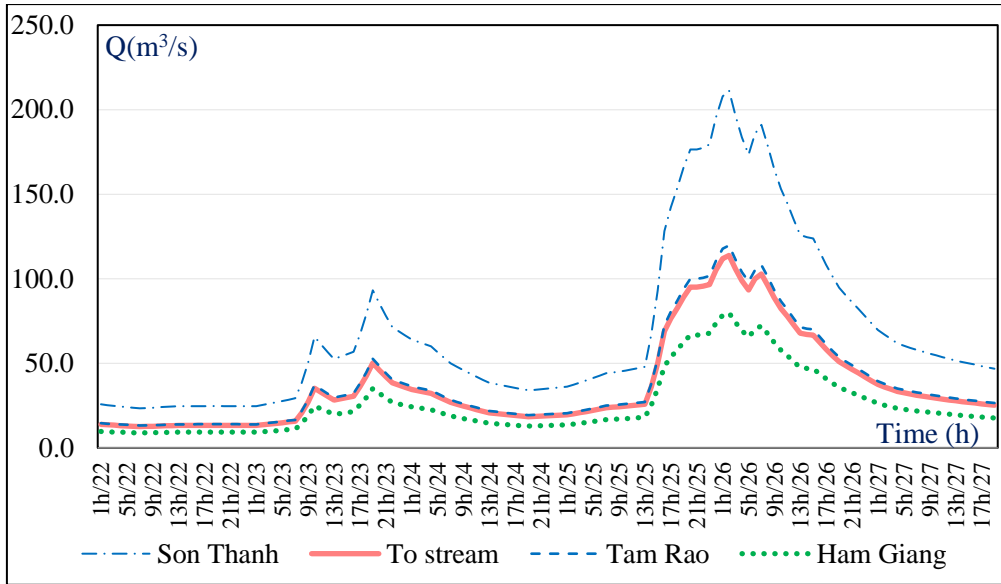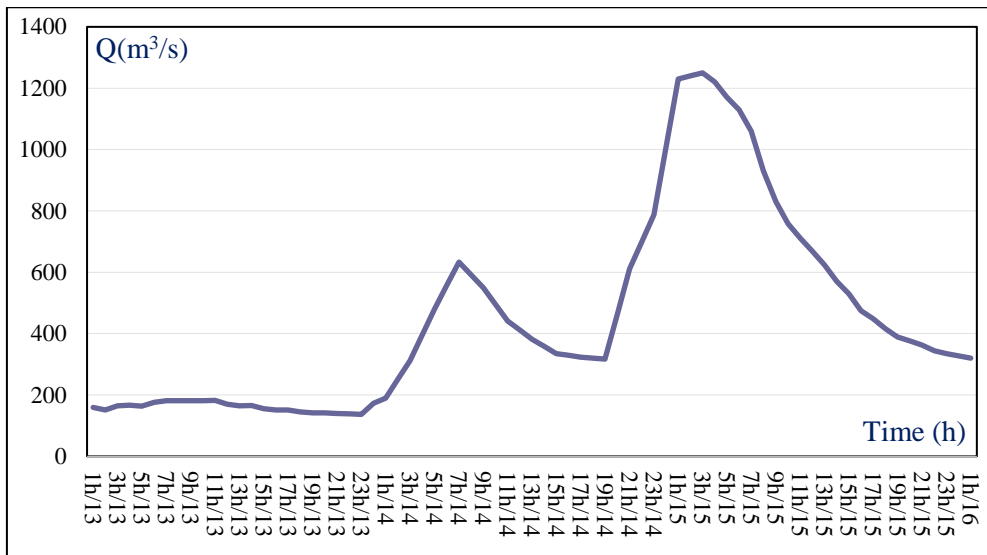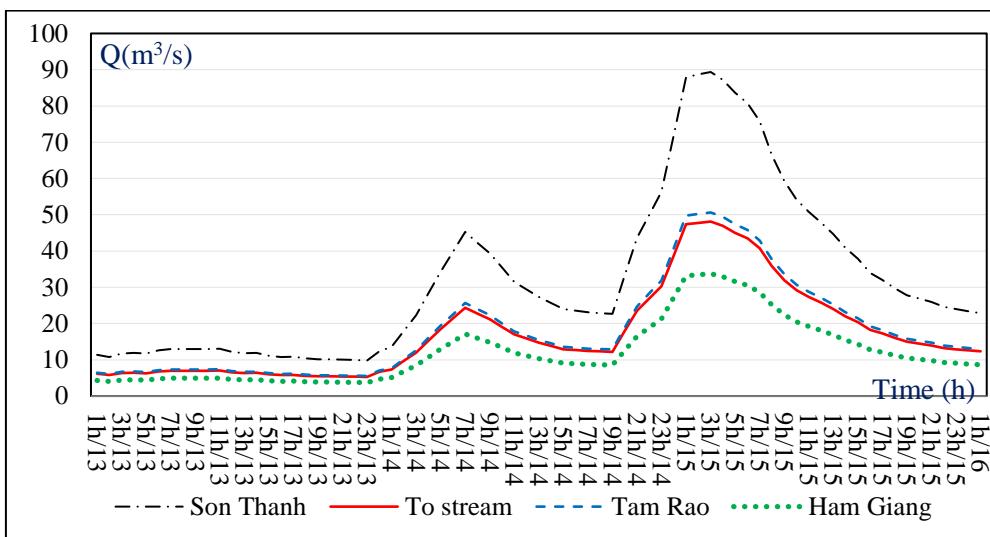

Figure 5. Discharge of Son Giang hydrological station from November 22 to 27, 2011.

Figure 6. Entry discharge from November 22 to 27, 2011.

Figure 7. Discharge of Son Giang hydrological station from September 13 to 15, 2013.


Figure 8. Discharge of Son Giang hydrological station from September 13 to 15, 2013.

3.2. One−dimensional kinematic wave model

The nonlinear one−dimensional kinematic wave model, after developing, is simulated and tested for flow on the Tra Khuc river from the Son Giang hydrological station to the outlet. The initial conditions are determined as follows: the width of the river sections is from 400 to 2200m, the slope of the river sections is from 1 to 5% and is calculated from the cross−sectional data. The hydraulic network is the same as the data in the Mike 11 model set up below, with a river length of 67,030m, 718 nodes, the simulation time step of 30 seconds, the input flow process at Son Giang station is shown in Figures 5 and 7, the Manning roughness coefficient for Tra Khuc River is from 0.032−0.037. The amount of accession to the middle zone at the tributaries of To, Son Thanh, Tam Rao and Ham Giang rivers is zoomed in proportion to the area with the Son Giang hydrological station shown in Figures 6 and 8.

The Nash indicator of the simulation results of the nonlinear one−dimensional dynamic wave model using at Tra Khuc hydrological station for the flood occurring from November 22 to 27, 2011 is 0.89. The similar simulation is implemented to the flood that occurred from 13 to 15 September 2013 which brings the Nash indicator of 0.85.
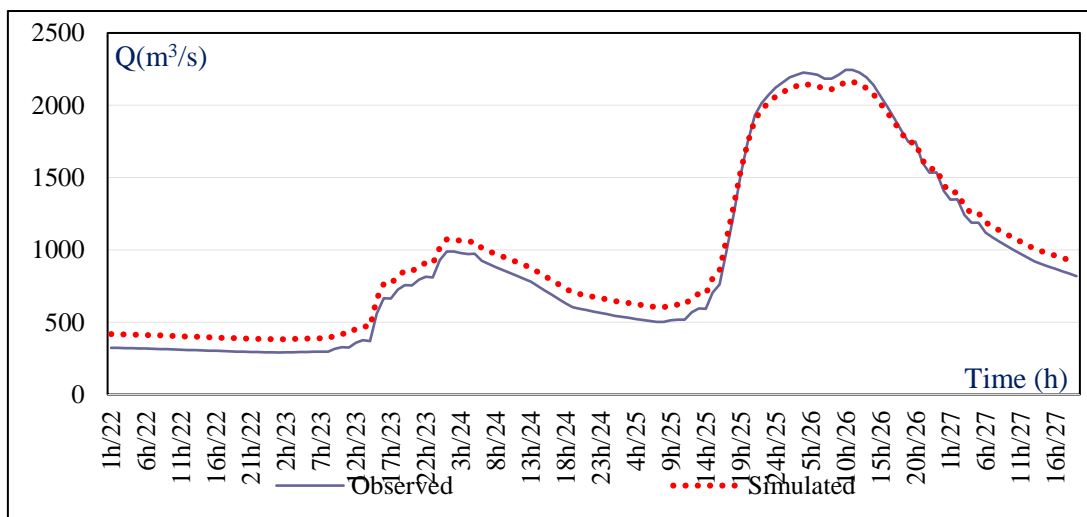

Figure 9. Observed and simulated discharge by kinematic wave model at Tra Khuc hydrological station during the flood from November 22 to 27, 2011.
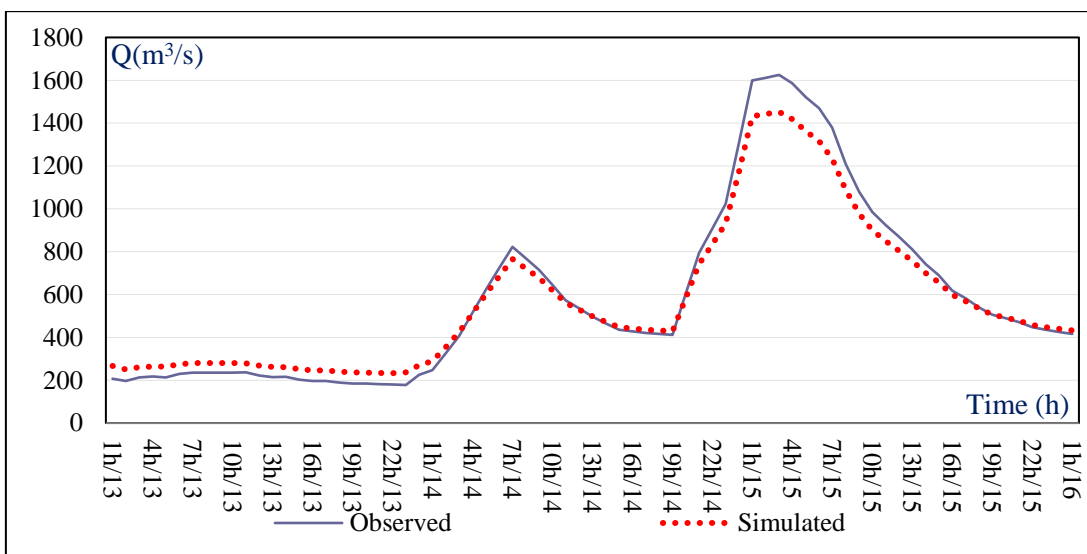

Figure 10. Observed and simulated discharge by kinematic wave model at Tra Khuc hydrological station during the flood from 13 to 15 September 2013.

3.3. Mike 11 model establishment

The river network is digitized from a map of 1/10,000 scale, using the Quang Ngai reference system with 108-degree zone 3 and then updating this reference system into the Mike 11 model. Updating 68 national standard elevation cross-sections from the Son Giang hydrological station to the sea mouth. The upper boundary is the discharge process curve at Son Giang which is shown in Figures 5 and 7, the lower boundary is the tidal water level process line. Joining the middle zone includes 4 tributaries of rivers and streams whose discharge process lines are shown in Figures 6 and 8. Manning roughness coefficients of sections on Tra Khuc river are determined as the Kinematic wave model established above, with values of river bed and banks from 0.032−0.037.
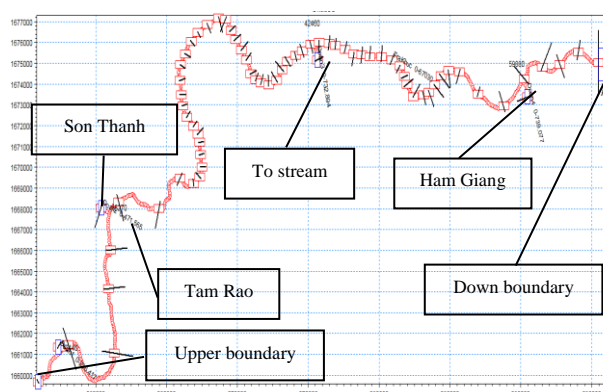
**Figure 11.** Hydraulic diagram at downstream of Tra Khuc river in Mike 11 model.
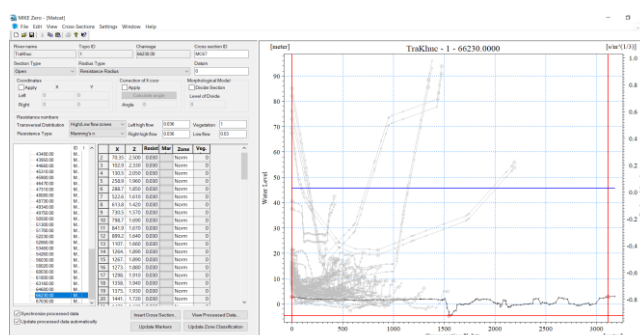
Figure 12. Cross section of Tra Khuc river downstream in Mike 11 model.

The tidal level is used as the lower boundary for the Mike 11 model, there is no tidal station at the outlet of Tra Khuc River, so the tidal boundary is determined by the tidal calculator in the Mike 21 Toolbox. The set of tidal parameters is taken from the parameter map of DHI with a resolution of $0.25^o \times 0.25^o$. The tidal water level at the mouth of Tra Khuc river is calculated at 15.15^oN and 108.94^oE, the parameters are shown in the figure 13 [3].

Number of		8	
	Name	**Amplitude**	**Phase**
1	M2	0.201654	77.01816
2	S2	0.0681956	109.7103
3	K1	0.3098014	187.8476
4	O1	0.292512	146.9564
5	N2	0.04320046	48.19024
6	P1	0.1018413	181.6318
7	K1	0.01616075	107.6501
8	Q1	0.05626091	153.8448

Figure 13. Tide model parameter set.
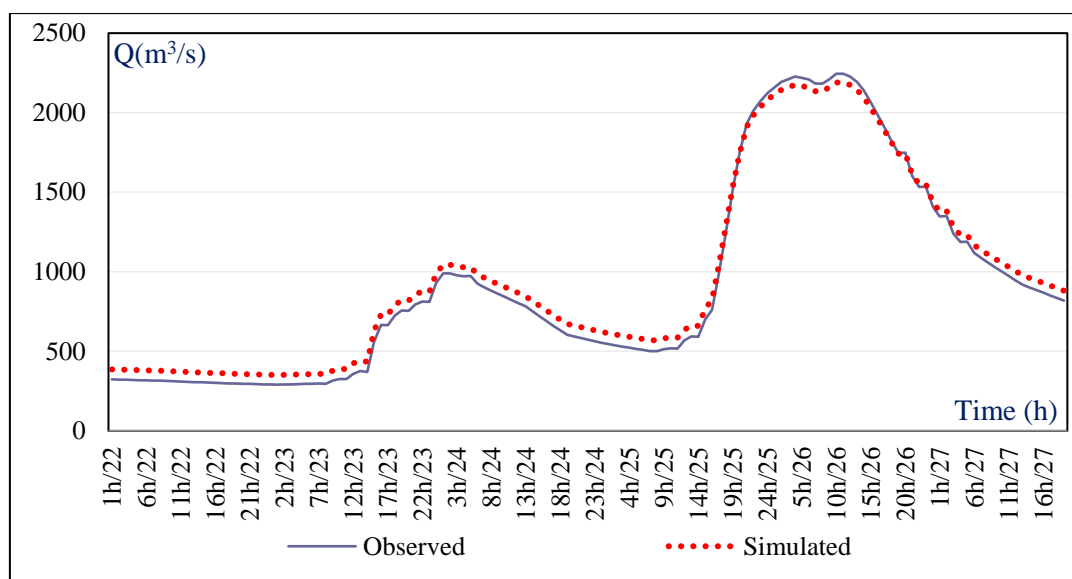
Figure 14. Observed and simulated discharge by Mike 11 model at Tra Khuc hydrological station during the flood from November 22 to 27, 2013.
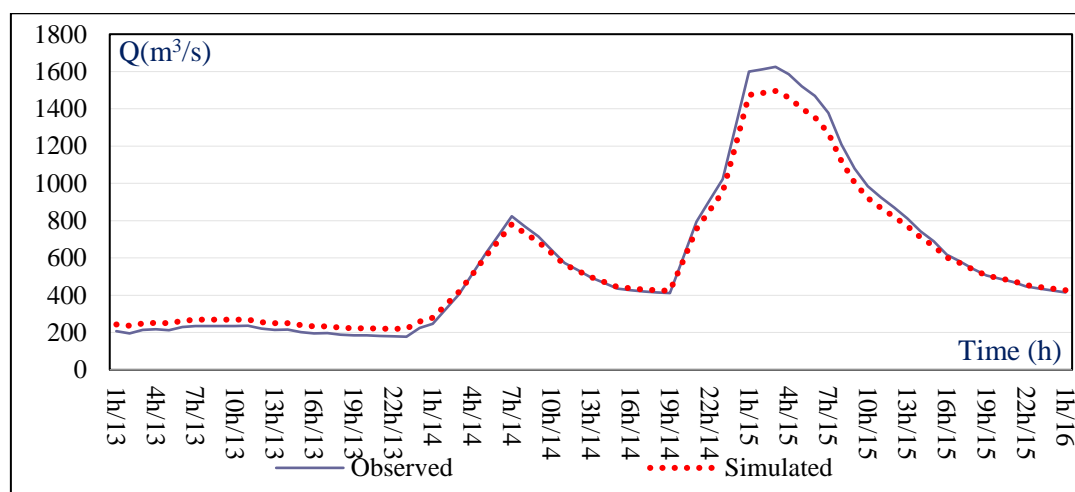
Figure 15. Observed and simulated discharge by Mike 11 model at Tra Khuc hydrological station during the flood from September 13 to 15, 2013.

The Mike 11 simulation results shows that the Nash indicator at Tra Khuc hydrological station for the flood occurring from November 22 to 27, 2011 is 0.92. Similarly, the Mike 11 simulation for the flood occurring from September 13 to 15, 2013 shows the Nash indicator of 0.88.

4. Conclusions

- The dynamic wave models has a wider simulation scope than the kinematic wave models but needs the cross-sectional data; therefore, applicability is limited in the absence of cross-sectional data.

- The simulation quality of the dynamic wave model is not much higher than that of the kinematic wave model in the upstream and middle flow where the lack of cross-sectional data is very common. Therefore, the Kinematic wave model is more likely to be applicable in mountainous rivers.

- The kinematic wave model does not use the lower boundary, so it is more suitable for forecasting than the dynamic wave model.

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